

The Fundamental Theorem of Invertible Matrices

Let A be an $n \times n$ matrix. The following statements are equivalent:

- a. A is invertible.
- b. $A\mathbf{x} = \mathbf{b}$ has a unique solution for every \mathbf{b} in \mathbb{R}^n .
- c. $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.
- d. The reduced row echelon form (RREF) of A is I_n .
- e. A is a product of elementary matrices.
- f. $\text{rank}(A) = n$
- g. $\text{nullity}(A) = 0$
- h. The column vectors of A are linearly independent.
- i. The column vectors of A span \mathbb{R}^n .
- j. The column vectors of A form a basis of \mathbb{R}^n .
- k. The row vectors of A are linearly independent.
- l. The row vectors of A span \mathbb{R}^n .
- m. The row vectors of A form a basis of \mathbb{R}^n .
- n. $\det A \neq 0$
- o. 0 is not an eigenvalue of A .