

## Tests of Convergence for Series

Test	Series	Converges	Diverges	Comment
$n$ th-Term	$\sum_{n=1}^{\infty} a_n$		$\lim_{n \rightarrow \infty} a_n \neq 0$	Can't test convergence
Geometric Series	$\sum_{n=0}^{\infty} ar^n$	$-1 < r < 1$	$ r  \geq 1$	Sum: $S = \frac{a}{1-r}$
Telescoping Series	$\sum_{n=1}^{\infty} (b_n - b_{n+1})$	$\lim_{n \rightarrow \infty} b_n = L$		Sum: $S = b_1 - L$
$p$ -Series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$p \leq 1$	Harmonic Series if $p = 1$
Absolute Convergence	$\sum_{n=1}^{\infty} a_n$	$\sum_{n=1}^{\infty}  a_n $ converges		
Alternating Series	$\sum_{n=1}^{\infty} (-1)^{n+1} a_n$	$0 < a_{n+1} \leq a_n$ and $\lim_{n \rightarrow \infty} a_n = 0$		$ R_N  \leq a_{N+1}$
Integral ( $f$ is continuous, positive, and decreasing)	$\sum_{n=1}^{\infty} f(n)$	$\int_1^{\infty} f(x) dx$ converges	$\int_1^{\infty} f(x) dx$ diverges	$0 < R_N \leq \int_N^{\infty} f(x) dx$
Ratio	$\sum_{n=1}^{\infty} a_n$	Absolutely if $\lim_{n \rightarrow \infty} \left  \frac{a_{n+1}}{a_n} \right  < 1$	$\lim_{n \rightarrow \infty} \left  \frac{a_{n+1}}{a_n} \right  > 1$	Inconclusive if $\lim_{n \rightarrow \infty} \left  \frac{a_{n+1}}{a_n} \right  = 1$
Root	$\sum_{n=1}^{\infty} a_n$	Absolutely if $\lim_{n \rightarrow \infty} \sqrt[n]{ a_n } < 1$	$\lim_{n \rightarrow \infty} \sqrt[n]{ a_n } > 1$	Inconclusive if $\lim_{n \rightarrow \infty} \sqrt[n]{ a_n } = 1$
Direct Comparison	$\sum_{n=1}^{\infty} a_n$	$0 < a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ converges	$0 < b_n \leq a_n$ and $\sum_{n=1}^{\infty} b_n$ diverges	
Limit Comparison ( $a_n, b_n > 0$ )	$\sum_{n=1}^{\infty} a_n$	$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ converges	$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ diverges	