Logical Simplifications

Consider the following line of code in the language Python.

```python
if x > 0 or (x <= 0 and y > 10):
```

If we let $p$ represent $x > 0$ and $q$ represent $y > 10$, then the condition in the above line of code can be expressed symbolically as

$$p \lor (\neg p \land q).$$

Using logical equivalences, we deduce the following.

$$p \lor (\neg p \land q) \iff (p \lor \neg p) \land (p \lor q) \quad \text{(from the distributive law)}$$

$$\iff T \land (p \lor q) \quad \text{(from the negation law)}$$

$$\iff p \lor q \quad \text{(from the identity law)}$$

A truth-table could also be used to show the equivalence.

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \lor (\neg p \land q)$</th>
<th>$p \lor q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
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<td>F</td>
<td>T</td>
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<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Therefore, the above line of code can be changed to

```python
if x > 0 or y > 10:
```

without changing its behavior.