

Logical Simplifications

Consider the following line of code in the language Python.

```
if x > 0 or (x <= 0 and y > 10):
```

If we let p represent $x > 0$ and q represent $y > 10$, then the condition in the above line of code can be expressed symbolically as

$$p \vee (\neg p \wedge q).$$

Using logical equivalences, we deduce the following.

$$\begin{aligned} p \vee (\neg p \wedge q) &\iff (p \vee \neg p) \wedge (p \vee q) && \text{(from the distributive law)} \\ &\iff \text{T} \wedge (p \vee q) && \text{(from the negation law)} \\ &\iff p \vee q && \text{(from the identity law)} \end{aligned}$$

A truth-table could also be used to show the equivalence.

p	q	$p \vee (\neg p \wedge q)$	$p \vee q$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

Therefore, the above line of code can be changed to

```
if x > 0 or y > 10:
```

without changing its behavior.