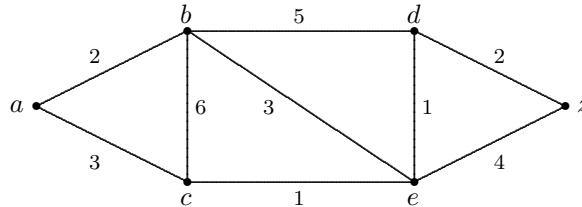


# Dijkstra's Algorithm

We consider a weighted connected simple graph  $G$  with vertices  $a = v_0, v_1, \dots, v_n = z$  and weights  $w(v_i, v_j) > 0$  where  $w(v_i, v_j) = \infty$  if  $\{v_i, v_j\}$  is not an edge. Dijkstra's algorithm finds the cost of the "cheapest" path between vertices  $a$  and  $z$ .

**procedure** *Dijkstra*( $G$ : weighted connected simple graph, with all weights positive)  
**for**  $i = 1$  **to**  $n$   
     $L(v_i) := \infty$   
 $L(a) := 0$   
 $S := \emptyset$   
**while**  $z \notin S$   
  **begin**  
     $u :=$  a vertex not in  $S$  with  $L(u)$  minimal  
     $S := S \cup \{u\}$   
    **for** all vertices  $v \notin S$   
      **if**  $L(u) + w(u, v) < L(v)$  **then**  $L(v) := L(u) + w(u, v)$   
  **end**  
**return**( $L(z)$ )

**Example:** Use Dijkstra's algorithm to find the cost of the cheapest path between  $a$  and  $z$  in the following weighted graph. Describe at each iteration the function  $L$  and set  $S$ .



*Solution:* The iterations of Dijkstra's algorithm are described in the following table.

$S$	$L(a)$	$L(b)$	$L(c)$	$L(d)$	$L(e)$	$L(z)$
$\emptyset$	0	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
$\{a\}$		2	3	$\infty$	$\infty$	$\infty$
$\{a, b\}$			3	7	5	$\infty$
$\{a, b, c\}$				7	4	$\infty$
$\{a, b, c, e\}$				5		8
$\{a, b, c, e, d\}$						7
$\{a, b, c, e, d, z\}$						

At the last iteration,  $z \in S$  and  $L(z) = 7$ . We conclude that the cheapest path from  $a$  to  $z$  has a cost of 7.