Tabular Method for Integration by parts

Example 1 Evaluate $\int x^2 \cos x \, dx$

D		I
x^2		$\cos x$
$2x$	$+$	$\sin x$
2	$-$	$-\cos x$
0	$+$	$-\sin x$

$$\begin{aligned} \int x^2 \cos x \, dx &= x^2 \sin x - 2x(-\cos x) + 2(-\sin x) + C \\ &= x^2 \sin x + 2x \cos x - 2 \sin x + C \end{aligned}$$

Example 2 Evaluate $\int (x^3 + 2x) \, e^{2x} \, dx$

D		I
$x^3 + 2x$		e^{2x}
$3x^2 + 2$	$+$	$e^{2x}/2$
$6x$	$-$	$e^{2x}/4$
6	$+$	$e^{2x}/8$
0	$-$	$e^{2x}/16$

$$\begin{aligned} \int (x^3 + 2x) \, e^{2x} \, dx &= \frac{x^3 + 2x}{2} \, e^{2x} - \frac{(3x^2 + 2)}{4} \, e^{2x} + \frac{6x}{8} \, e^{2x} - \frac{6}{16} \, e^{2x} + C \\ &= \frac{e^{2x}}{8}(4x^3 - 6x^2 + 14x - 7) + C \end{aligned}$$

Gilles Cazelais. Typeset with LaTeX on April 20, 2006.

Example 3 Evaluate $\int x^3 \ln x \, dx$

D		I
$\ln x$	$+$ ↘	x^3
$1/x$	← $-$	$x^4/4$

$$\begin{aligned}\int x^3 \ln x \, dx &= \frac{x^4}{4} \ln x - \int \left(\frac{x^4}{4}\right)\left(\frac{1}{x}\right) dx \\ &= \frac{x^4}{4} \ln x - \frac{1}{4} \int x^3 \, dx \\ &= \frac{x^4}{4} \ln x - \frac{x^4}{16} + C\end{aligned}$$

Example 4 Evaluate $\int e^{2x} \cos x \, dx$

D		I
e^{2x}	$+$ ↘	$\cos x$
$2e^{2x}$	$-$ ↘	$\sin x$
$4e^{2x}$	← $+$	$-\cos x$

$$\begin{aligned}\int e^{2x} \cos x \, dx &= e^{2x} \sin x - 2e^{2x}(-\cos x) + \int 4e^{2x}(-\cos x) \, dx \\ &= e^{2x} \sin x + 2e^{2x} \cos x - 4 \int e^{2x} \cos x \, dx\end{aligned}$$

Observe that $\int e^{2x} \cos x \, dx$ appears on both sides of the last equation. Therefore,

$$5 \int e^{2x} \cos x \, dx = e^{2x} \sin x + 2e^{2x} \cos x$$

We get the final answer by dividing by 5 and by introducing the constant of integration.

$$\int e^{2x} \cos x \, dx = \frac{e^{2x}}{5}(\sin x + 2 \cos x) + C$$