Integration by parts

The formula for integration by parts is the following.

$$\boxed{\int u\,dv = uv - \int v\,du}$$

Let's use it to evaluate $\int x\cos x\,dx$.

We choose

$$u = x \quad \text{and} \quad dv = \cos x\,dx$$

By differentiating u and integrating dv, we obtain

$$du = dx \quad \text{and} \quad v = \sin x$$

Therefore,

$$\begin{aligned}\int x\cos x\,dx &= x\sin x - \int \sin x\,dx \\ &= x\sin x + \cos x + C\end{aligned}$$

Evaluating integrals such as

$$\int x^n e^{ax}\,dx, \quad \int x^n \cos(ax)\,dx, \quad \text{or} \quad \int x^n \sin(ax)\,dx$$

for a positive integer n, would require using integration by parts n times. For cases like these, the Tabular Method can help us organize our work.

To illustrate the Tabular Method let's evaluate $\int x^2 e^{2x}\,dx$.

$$\begin{array}{ccc} D & & I \\ \hline x^2 & & e^{2x} \\ & + & \\ 2x & & e^{2x}/2 \\ & - & \\ 2 & & e^{2x}/4 \\ & + & \\ 0 & & e^{2x}/8 \end{array}$$

We differentiate the entries in the D column until we get a zero. We integrate the entries in the I column. The signs along the diagonal arrows always alternate: $+, -, +, \cdots$. The answer is obtained by adding the product of the diagonal entries with the appropriate sign.

$$\begin{aligned}\int x^2 e^{2x}\,dx &= \frac{x^2}{2}e^{2x} - \frac{2x}{4}e^{2x} + \frac{2}{8}e^{2x} + C \\ &= e^{2x}\left(\frac{x^2}{2} - \frac{x}{2} + \frac{1}{4}\right) + C\end{aligned}$$

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