Graph of $y = a \sin(bx + c)$

The graph of a basic sine curve $y = \sin x$ is shown below.

For a basic sine curve, one cycle corresponds to $0 \leq x \leq 2\pi$, the period (i.e., the length of one cycle) is $2\pi$, and the amplitude is 1.

Let’s consider $y = a \sin x$. The effect of $a$ is to stretch the graph of $y = \sin x$ by a factor $|a|$. If $a < 0$, there is also a reflection through the $x$-axis.

For $y = a \sin x$, the amplitude is $|a|$ and the period is $2\pi$.

Let’s consider $y = a \sin(bx)$. We assume that $b > 0$ without loss of generality. If $b < 0$, we can use the identity $\sin(-\theta) = -\sin \theta$ to reduce to a case where $b > 0$. One cycle of $y = a \sin(bx)$ is obtained as follows.

$$0 \leq bx \leq 2\pi \implies 0 \leq x \leq \frac{2\pi}{b}$$

The length of one cycle is $\frac{2\pi}{b} - 0 = \frac{2\pi}{b}$, therefore the period is $\frac{2\pi}{b}$.

For $y = a \sin(bx)$, the amplitude is $|a|$ and the period is $\frac{2\pi}{b}$.
Let’s now consider $y = a \sin(bx + c)$. One cycle of this sine curve is obtained as follows.

$$0 \leq bx + c \leq 2\pi \implies -c \leq bx \leq 2\pi - c \implies \frac{-c}{b} \leq x \leq \frac{2\pi}{b} - \frac{c}{b}$$

We see that the length of one cycle is $(\frac{2\pi}{b} - \frac{c}{b}) - (-\frac{c}{b}) = \frac{2\pi}{b}$, therefore the period is $\frac{2\pi}{b}$. Observe that $c$ does not affect the period.

If we compare the cycle $0 \leq x \leq \frac{2\pi}{b}$ (when $c = 0$) with the cycle $-\frac{c}{b} \leq x \leq \frac{2\pi}{b} - \frac{c}{b}$, we see that there is a shift of $-\frac{c}{b}$. The quantity $-\frac{c}{b}$ is called the phase shift.

If $c > 0$, we get a shift to the left.

If $c < 0$, we get a shift to the right.

For $y = a \sin(bx + c)$, the amplitude is $|a|$, the period is $\frac{2\pi}{b}$ and the phase shift is $-\frac{c}{b}$.

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