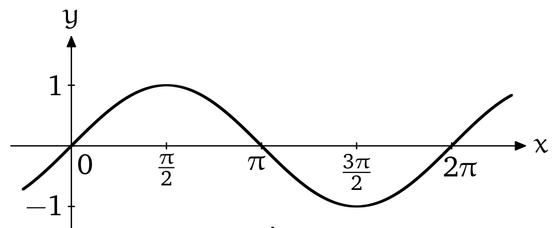
Graph of $y = a\sin(bx + c)$

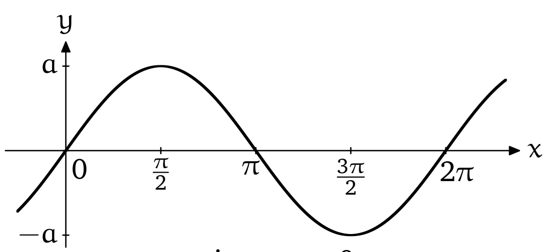
The graph of a basic sine curve $y = \sin x$ is shown below.



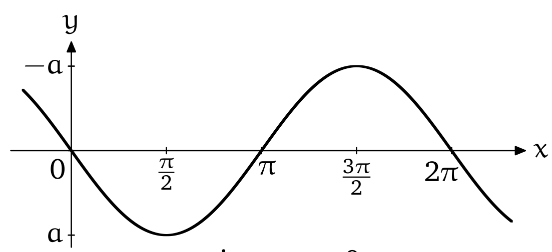
$y = \sin x$

For a basic sine curve, one **cycle** corresponds to $0 \leqslant x \leqslant 2\pi$, the **period** (i.e., the length of one cycle) is 2π, and the **amplitude** is 1.

Let's consider $y = a\sin x$. The effect of a is to stretch the graph of $y = \sin x$ by a factor $|a|$. If $a < 0$, there is also a reflection through the x-axis.



$y = a\sin x,\ \ a > 0$



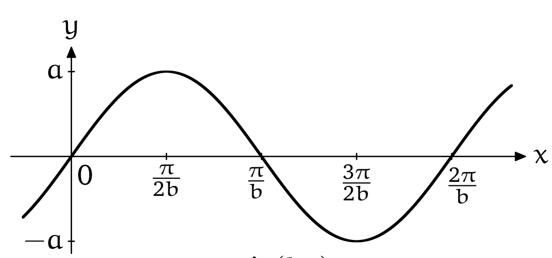
$y = a\sin x,\ \ a < 0$

For $y = a\sin x$, the **amplitude** is $|a|$ and the **period** is 2π.

Let's consider $y = a\sin(bx)$. We assume that $b > 0$ without loss of generality. If $b < 0$, we can use the identity $\sin(-\theta) = -\sin\theta$ to reduce to a case where $b > 0$. One cycle of $y = a\sin(bx)$ is obtained as follows.

$$0 \leqslant bx \leqslant 2\pi \implies 0 \leqslant x \leqslant \frac{2\pi}{b}$$

The length of one cycle is $\frac{2\pi}{b} - 0 = \frac{2\pi}{b}$, therefore the period is $\frac{2\pi}{b}$.



$y = a\sin(bx)$

For $y = a\sin(bx)$, the **amplitude** is $|a|$ and the **period** is $\frac{2\pi}{b}$.

Gilles Cazelais. Typeset with LaTeX on February 28, 2007.

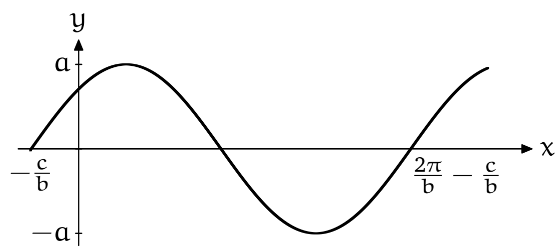
Let's now consider $y = a\sin(bx + c)$. One cycle of this sine curve is obtained as follows.

$$0 \leqslant bx + c \leqslant 2\pi \implies -c \leqslant bx \leqslant 2\pi - c \implies \boxed{-\frac{c}{b} \leqslant x \leqslant \frac{2\pi}{b} - \frac{c}{b}}$$

We see that the length of one cycle is $\left(\frac{2\pi}{b} - \frac{c}{b}\right) - \left(-\frac{c}{b}\right) = \frac{2\pi}{b}$, therefore the period is $\frac{2\pi}{b}$. Observe that c does not affect the period.

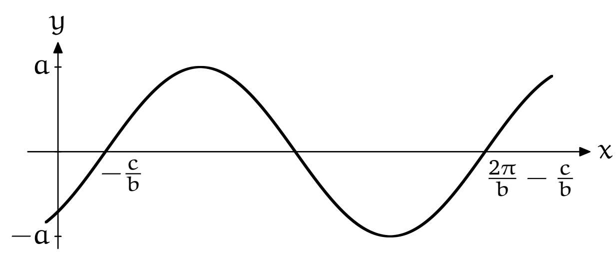
If we compare the cycle $0 \leqslant x \leqslant \frac{2\pi}{b}$ (when $c = 0$) with the cycle $-\frac{c}{b} \leqslant x \leqslant \frac{2\pi}{b} - \frac{c}{b}$, we see that there is a shift of $-\frac{c}{b}$. The quantity $-\frac{c}{b}$ is called the **phase shift**.

If $c > 0$, we get a shift to the left.



$y = a\sin(bx + c),\ c > 0$

If $c < 0$, we get a shift to the right.



$y = a\sin(bx + c),\ c < 0$

For $y = a\sin(bx + c)$, the **amplitude** is $|a|$, the **period** is $\frac{2\pi}{b}$ and the **phase shift** is $-\frac{c}{b}$.

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